

Introduction

The cascaded integrator-comb (CIC) filter is a class of hardware-efficient linear phase finite impulse response (FIR) digital filters. CIC filters achieve sampling rate decrease (decimation) and sampling rate increase (interpolation) without using multipliers. Altera's CIC Compiler MegaCore® function implements various CIC filters based on Hogenauer's method.



CIC filters were first proposed by Eugene Hogenauer in 1981, For more information about CIC filters, refer to Eugene B. Hogenauer, "An economical class of digital filters for decimation and interpolation," *IEEE Transactions on Acoustics, Speech and Signal Processing*, pp. 155-162, April 1981.

A CIC filter consists of an equal number of stages of ideal integrator filters and comb filters. Its frequency response may be tuned by selecting the appropriate number of cascaded integrator and comb filter pairs. The highly symmetric structure of a CIC filter allows efficient implementation in hardware. However, the disadvantage of a CIC filter is that its pass band is not flat, which is undesirable in many applications. Fortunately, this problem can be alleviated by a compensation filter.

This application note presents theory and methods for designing CIC compensating filters for sample rate conversion systems. The MATLAB Signal Processing Toolbox is used to design the coefficients of the compensating FIR filters. This application note also describes how to choose parameters for designing a compensation filter and then implements an example decimation system using the Altera® CIC Compiler MegaCore function and the FIR Compiler MegaCore function.

The following topics are discussed in this document:

- ["Prerequisites" on page 2](#)
- ["CIC Filter Structure" on page 2](#)
- ["CIC Compensation Filter Design" on page 4](#)
- ["Data Rate Down Conversion Example" on page 11](#)
- ["Conclusion" on page 17](#)

Prerequisites

This document targets digital signal processing (DSP) systems engineers who must design CIC compensation filters for rate conversion systems.

A basic knowledge of DSP and digital filter design will help you understand the trade-off between various CIC compensation filter design methods.

In addition, to understand and duplicate the examples and figures used in this application note, you should have the following:

- Some experience with MATLAB and SIMULINK
- Some knowledge of Altera DSP Solutions, including DSP Builder



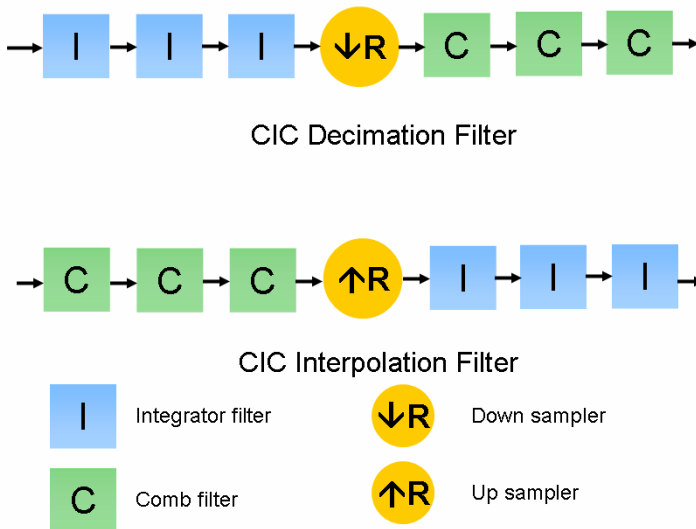
The design example used in this application note can be found at:

www.altera.com/support/examples/dsp-builder/exm-digital-down-conv-cic-fir.html

CIC Filter Structure

The basic elements of a CIC filter are integrator filters and comb filters, as shown in Figure 1.

Figure 1. Block Diagram of Three-Stage CIC Decimation and Interpolation Filters



An integrator filter is a single pole accumulator with a transfer function $H_I(z)$ (Equation 1):

$$(1) \quad H_I(z) = \frac{1}{1 - z^{-1}}$$

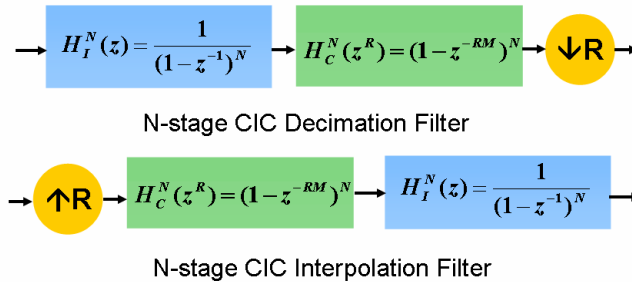
A comb filter is a differentiator with a transfer function $H_C(z)$ (Equation 2):

$$(2) \quad H_C(z) = 1 - z^{-M}$$

In this equation, M is the differential delay, and is usually limited to 1 or 2.

In a CIC filter, the integrators operate at high sampling frequency (f_s), and the comb filters operate at low frequency (f_s/R). Using the Noble identities, the equivalent frequency response of their cascade can be calculated (Figure 2).

Figure 2. Block Diagram of the Equivalent Frequency Response of an N-Stage CIC Filter



Equation 3 shows the total response of a CIC filter at high frequency (f_s):

$$(3) \quad H(z) = H_I^N(z)H_C^N(z^R) = \left(\sum_{k=0}^{RM-1} z^{-k} \right)^N$$

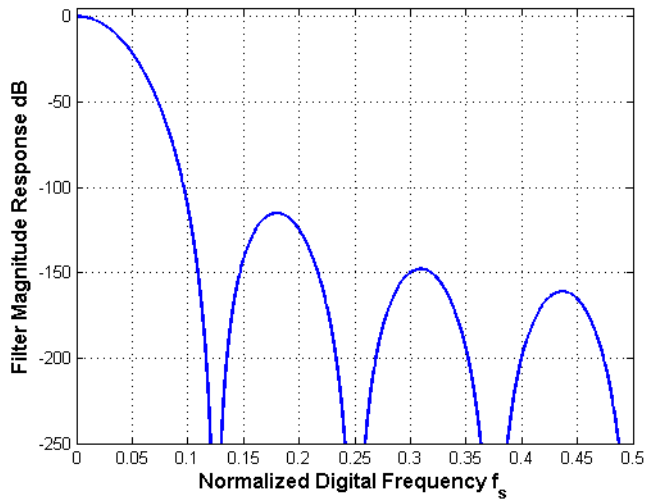
In this equation, N is the number of integrator-comb filter pairs, and R is the rate change factor. Equation 3 implies that the equivalent time domain impulse response of a CIC filter can be viewed as a cascade of N rectangular pulses. Each rectangular pulse has RM taps.

Equation 4 shows the magnitude response of an N -stage CIC filter at high frequency (f_s):

$$(4) \quad |H(f)| = \left| \frac{\sin(\pi M f)}{\sin\left(\frac{\pi f}{R}\right)} \right|^N$$

Figure 3 shows an example of a CIC filter magnitude response:

Figure 3. Magnitude Response of a CIC Filter with $N = 9$, $R = 8$, and $M = 1$



For more information about CIC filters, refer to Matthew Donadio, *Cascaded Integrator-Comb (CIC) Filter Introduction*, available at www.dspguru.com/info/tutor/cic.htm.

CIC Compensation Filter Design

Figure 3 shows that when the number of stages is large, the CIC filter frequency response does not have a wide, flat pass band. To overcome the magnitude droop, a FIR filter that has a magnitude response that is the inverse of the CIC filter can be applied to achieve frequency response correction. Such filters are called “compensation filters.”

For data rate down conversion, the compensation filter follows the CIC filter. For up sampling systems, the compensation FIR filter pre-conditions the data and is followed by a CIC filter. In other words, the compensation filter always operates at the lower rate in a rate conversion

design. One benefit of running the compensation filter at the low rate is to achieve a more efficient hardware solution, that is, more time sharing in the compensation FIR filter.

Equation 4 gives the magnitude response of a CIC filter. To achieve a flat pass band, the compensation FIR filter should have a magnitude response that is the inverse of Equation 4, as shown in Equation 5:

$$(5) \quad G(f) = \left| MR \left(\frac{\sin(\pi f / R)}{\sin(\pi Mf)} \right) \right|^N \approx \left| \frac{\pi Mf}{\sin(\pi Mf)} \right|^N = |\text{sinc}^{-1}(Mf)|^N$$

When R is large, the compensation filter response can be approximated by the inverse sinc function, so the compensation filter is sometimes referred to as the “inverse sinc filter.”

In some rate change systems, compensation filters are also multirate filters. They can implement additional decimation or interpolation as necessary, but usually by a factor of 2 or less.

Generating Compensation Filter Coefficients Using MATLAB

Any filter design tool that generates filter coefficients based on the specified frequency response can be used to design a CIC compensation filter. In this application note, the MATLAB Signal Processing Toolbox function `fir2` is used to generate the coefficients for CIC compensation filters.

The `fir2` function designs FIR filters with an arbitrary frequency response based on the frequency sampling method. The generated filter coefficients are real and symmetric. You can specify a digital FIR filter order L with the frequency response specified by vectors F (frequency) and A (magnitude response on break points F). The `fir2` function returns an $(L+1)$ vector of filter coefficients.



The MATLAB script provided in this application note is automatically generated by the Altera CIC Compiler MegaCore function, version 7.1.



For more information about the frequency sampling method, refer to L.R. Rabiner, B. Gold, and C.A. McGonegal, “An approach to the approximation problem for nonrecursive digital filters,” IEEE Transactions on Audio and Electroacoustics, pp. 83-106, vol. Au-18, No. 2, June 1970.



For more information about the `fir2` function, refer to the MATLAB Help.

Example 1 shows a MATLAB script that generates the CIC compensation filter coefficients:

Example 1. Generating the CIC Compensation Filter Coefficients

```

##### CIC filter parameters #####
R = 4; % Decimation factor
M = 1; % Differential delay
N = 8; % Number of stages
B = 18; % Coeffi. Bit-width
Fs = 91.392e6; % (High) Sampling freq in Hz before decimation
Fc = 4.85e6; % Pass band edge in Hz

##### fir2.m parameters #####
L = 110; % Filter order; must be even
Fo = R*Fc/Fs; % Normalized Cutoff freq; 0<Fo<=0.5/M;
% Fo = 0.5/M; % use Fo=0.5 if you don't care responses are
% outside the pass band

##### CIC Compensator Design using fir2.m #####
p = 2e3; % Granularity
s = 0.25/p; % Step size
fp = [0:s:Fo]; % Pass band frequency samples
fs = (Fo+s):s:0.5; % Stop band frequency samples
f = [fp fs]*2; % Normalized frequency samples; 0<=f<=1
Mp = ones(1,length(fp)); % Pass band response; Mp(1)=1
Mf(2:end) = abs( M*R*sin(pi*fp(2:end)/R) ./sin(pi*M*fp(2:end))) .^N;
Mf = [Mp zeros(1,length(fs))];
f(end) = 1;
h = fir2(L,f,Mf); % Filter length L+1
h = h/max(h); % Floating point coefficients
hz = round(h*power(2,B-1)-1); % Fixed point coefficients

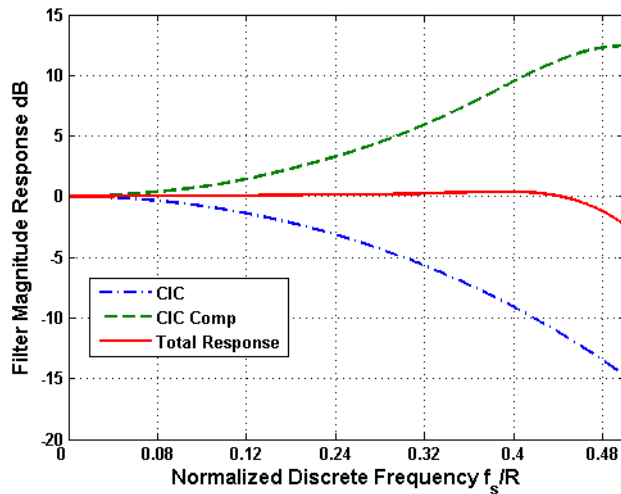
```

Compensation Filter Example

Figure 4 shows a simple example of a CIC filter response and its compensation filter response. The blue dotted line is the magnitude response of a CIC filter with rate change factor $R = 4$, differential delay $M = 1$, and number of integrator-comb filter pairs $N = 4$. The response is plotted at low frequency f_s/R . The green dashed curve is the single rate compensation filter response plotted over its operating frequency, f_s/R . The product of the dotted curve and the dashed curve is the total response of the CIC and compensation filter cascade, represented by the red solid curve.

The filter cascade clearly has a reasonably flat response. In this example, a 15-tap FIR compensation filter is designed using `fir2`. Its 18-bit wide coefficients are = [-215, 446, -1258, 3213, -7586, 17668, -44268, 131071, -44268, 17668, -7586, 3213, -1258, 446, -215]. The overall response plotted at the high frequency f_s is shown in Figure 5 on page 8.

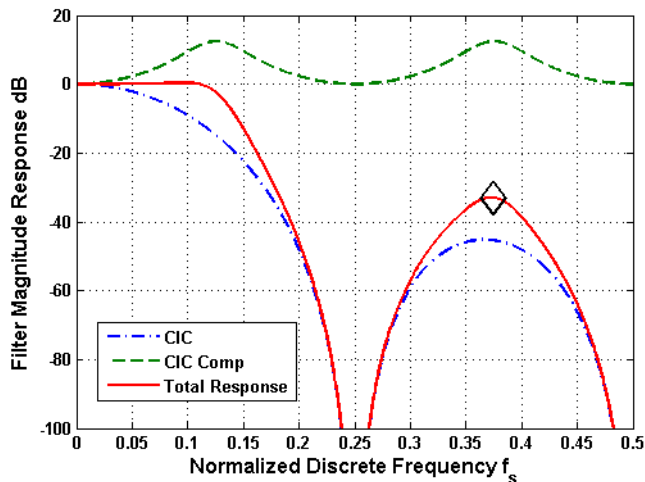
Figure 4. Compensation Filter Response for a 4-Stage CIC Filter, Plotted Over f_s/R



Note that the compensation filter operates at low frequency (f_c/R). For single rate compensation filters to avoid aliasing, the cutoff frequency (f_c) is, at most, half its output frequency: $f_c \leq (f_s/R)/2$. When the cutoff frequency is exactly $(f_s/R)/2$, as the case shown in Figure 4, the compensation filter has an inverse sinc response across its entire bandwidth, thus it is also called a “wideband compensation filter” (Figure 5).

In a multirate compensation filter, a decimation compensation filter with a rate change factor of 2 has input sampling rate f_s/R and output sampling rate $(f_s/R)/2$. To avoid aliasing, the compensation filter must have a cutoff frequency that is no more than half of $(f_s/R)/2$, that is, $(f_s/R)/4$.

Figure 5. Wideband Compensation Filter Response Over f_s for $R = 4$, $N = 4$, and $M = 1$ Note (1)



Note to Figure 5

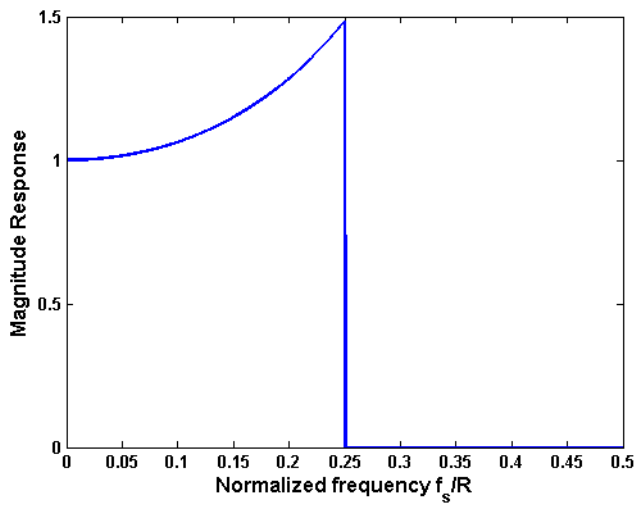
- (1) The black diamond marker indicates stop band amplification due to the compensation filter frequency response correction.

Choosing Pass Band Edge

Narrow transition bandwidth and good stop band attenuation are desirable properties of a narrow band rate conversion system. Unfortunately, a CIC filter alone does not offer these properties. As demonstrated in Figure 4, in addition to lacking a flat pass band, the CIC filter by itself also suffers from the lack of a well-defined transition band. This problem can be alleviated by controlling the compensation filter response. Instead of having the wide band compensation response, as demonstrated in Figure 4 and Figure 5, you can impose constraints on the low pass compensation filter, such as pass band edge and stop band attenuation.

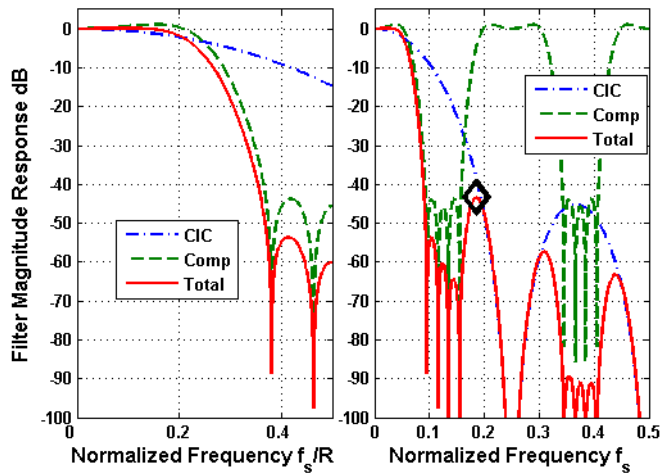
For example, you can modify the requirement of the compensation filter example (refer to “Compensation Filter Example” on page 6). Instead of having wide band compensation, a pass band edge is imposed at a quarter of the normalized low frequency f_s/R : $f_c = (f_s/R)/4$. The ideal response of the compensation filter is inverse sinc within the pass band, and zero outside the pass band, as shown in Figure 6.

Figure 6. Ideal Compensation Filter Response with a Cutoff Frequency of $(f_s/R)/4$



Based on the ideal frequency response, the new compensation filter coefficients can be generated using `fir2`. The responses are plotted in [Figure 7](#).

Figure 7. Designed Compensation Filter Response with a Normalized Cutoff Frequency of $(f_s/R)/4$ Note (1)



Note to Figure 7

- (1) The black diamond marker indicates stop band amplification due to the compensation filter frequency response correction.

Figures 5 and 7 also demonstrate the impact of choosing pass band width on noise amplification. In these examples, the CIC filter has its first null at $(1/M)/R$ on the high frequency scale f_s (that is, the first null occurs at $1/M$ at low frequency f_s/R). As the pass band edge gets closer to the null, the CIC filter attenuates more and the compensation filter needs to provide more correction. This correction causes noise amplification in the stop band. The black diamond markers in Figures 5 and 7 illustrate this point. The total response in the stop band has a spike at images of the cutoff frequency. The closer the pass band edge is to the first null and the more stages there are, the higher the spike is. In the extreme case—the wide band compensation case shown in Figure 5—the noise amplification can be significant. Therefore, a careful choice of pass band width is important when designing the CIC filter and compensation filter cascade.



A good practice is to choose the pass band edge to be less than a quarter of the first null on the low frequency scale f_s/R .

Data Rate Down Conversion Example



This section contains instructions for designing a practical decimation filter chain based on the IEEE 802.16d (WiMAX) digital down conversion (DDC) requirement. The WiMAX DDC function has a tight system requirement for frequency responses and is a good example of how to design a multi-rate system and choose filter parameters.

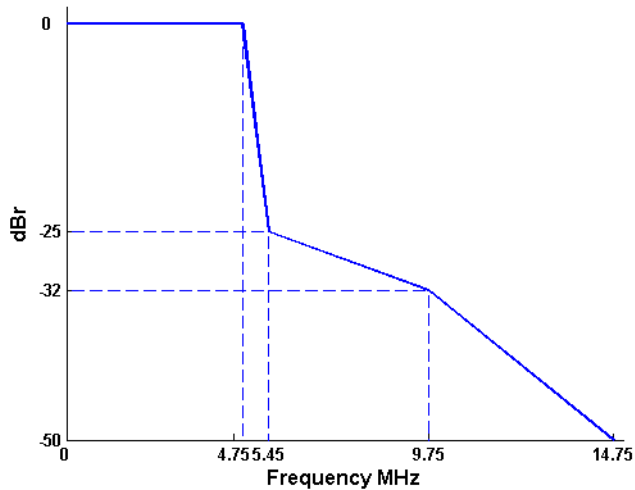
For more information about the IEEE standard, refer to *IEEE Standard for Local and Metropolitan Area Networks, Part 16: Air Interface for Fixed Broadband Wireless Access Systems*, IEEE P802.16-REVd/D5-2004, May 2004.

The design uses a decimation-by-8 filter chain. [Table 1](#) lists the key parameters of the system.

Table 1. WiMAX DDC Example Total Spectrum Requirement	
Parameter	Value
Input Sampling Frequency	91.392 MHz
Output Sampling Frequency	11.424 MHz
Pass Band Edge	4.75 MHz
Pass Band Ripple	< 0.05 dB
Stop Band Attenuation	> 90 dB

The overall filter response follows the WiMAX downlink spectrum mask, as shown in [Figure 8](#).

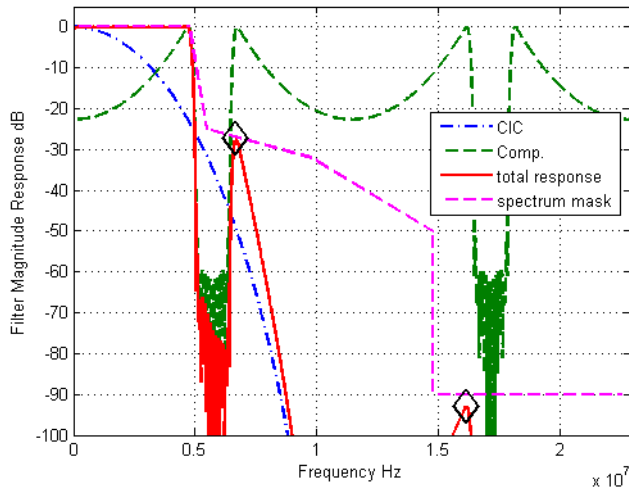
Figure 8. WiMAX Transmit Spectrum Mask



Solution 1: Single Stage Decimation

In this solution, the decimation-by-8 CIC filter implements the rate change with differential delay $M = 1$ and number of stages $N = 9$ (to achieve the stop band attenuation). The compensation filter is a single rate FIR filter operating at 11.424 MHz. Its pass band edge is chosen to be 4.85 MHz, slightly larger than the required pass band edge 4.75 MHz to guarantee good performance for data subcarriers at 4.75 MHz. The filter order, L , is chosen to be 110. The large filter order is necessary to meet the small pass band ripple requirement and the narrow transition bandwidth requirement. Using `fir2` and 18-bit width to represent filter coefficients, the designed CIC filter and compensation filter cascade has the response shown in [Figure 9](#). The performance is summarized in [Table 2](#) on [page 16](#).

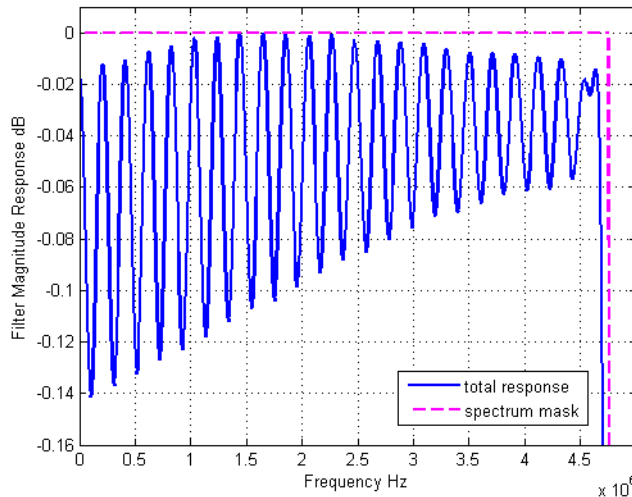
Figure 9. CIC and Compensation Filter Responses with WiMAX Spectrum Mask for Single Stage Decimation Design *Note (1)*



Note to Figure 9

- (1) The black diamond markers indicate stop band amplification due to the compensation filter frequency response correction

The noise amplification due to compensation filter frequency correction is evident in the above design, as illustrated by the black diamond markers. The overall frequency response appears to comply with the WiMAX spectrum mask. However, a closer look at the pass band reveals that the pass band ripple exceeds the allowed variation of 0.05 dB, as shown in Figure 10.

Figure 10. Pass Band of Designed Signal Stage Decimation Filter Chain

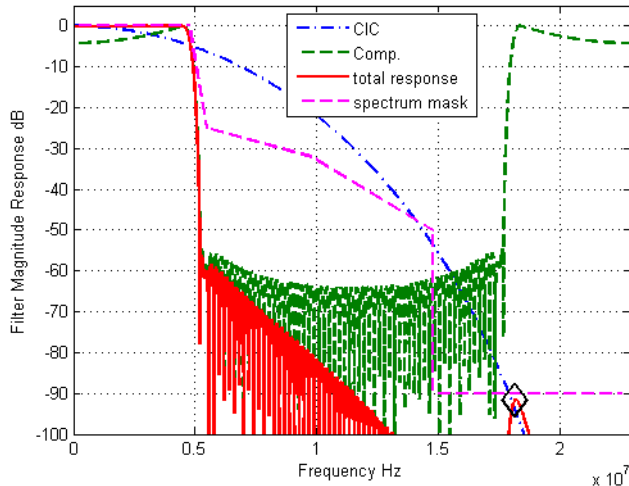
Increasing the compensation filter order L improves the pass band variation slightly, but not enough to meet system requirements. On the other hand, a high compensation filter order is not only impractical, but also causes large noise amplification in the transition band. In this solution setup, when L exceeds 160, the magnitude response around 6.6 MHz (demonstrated by the first black diamond marker in [Figure 9](#)) can exceed the WiMAX spectrum mask. As the last column of [Table 2 on page 16](#) demonstrates, in this single decimation stage solution, the normalized pass band edge is 0.42 on the low frequency scale (f_s/R), which is too close to the first null. The recommended normalized digital pass band width is 0.25.

Solution 2: Multi-Stage Decimation

Digital filter design theory states that with all other parameters fixed, the pass band variation is proportional to the normalized transition bandwidth. The normalized transition bandwidth in this example is determined by the WiMAX spectrum mask and the operating frequency of the compensation filter. The single stage decimation architecture was not able to meet the filter design requirement for two reasons: 1) the operating frequency of the compensation filter is too low, causing a wide normalized transition bandwidth, and 2) the normalized cutoff frequency is too close to the first null.

The natural solution to these problems is to divide the decimation into multiple stages. In the second solution, a decimation-by-4 CIC filter is followed by a compensation FIR filter that implements additional decimation-by-2. The CIC filter has $N = 8$ and the differential delay, M , is 1. The compensation filter input has a sample frequency of 22.848 MHz and output signals at 11.424 MHz. The compensation filter, L , has an order of 110, and the pass band edge is still 4.85 MHz. Compared with Solution 1, the normalized transition bandwidth is halved, and the normalized pass band edge is also halved. The performance parameters are summarized in Table 2 on page 16. The response of the filter cascade is shown in Figures 11 and 12.

Figure 11. CIC and Compensation Filter Responses with WiMAX Spectrum Mask for Multi-Stage Decimation Design Note (1)



Note to Figure 11

- (1) The black diamond marker indicates stop band amplification due to the compensation filter frequency response correction

The noise amplification in the multi-stage decimation solution is much milder than in the single stage decimation, due to reduced normalized cutoff frequency. The black diamond marker in Figure 11 marks the frequency response spike caused by the image of the compensation filter pass band edge.

A closer look of the pass band confirms that the pass band variation requirement in the multi-stage decimation setup has been met. Figure 12 shows the pass band responses of Solution 2.

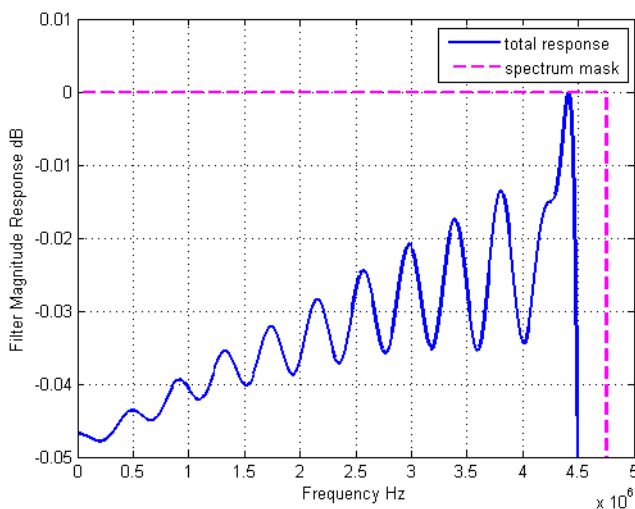
Figure 12. Pass Band Responses of the Multi-Stage Design


Table 2 shows the WiMAX DDC filter design performances.

Table 2. WiMAX DDC Filter Design Performances *Note (1)*

Cases	Filter Order	Pass Band Ripple (dB)	Magnitude at Image of Pass Band Edge (dB)	Stop Band Attenuation (dB)	Pass Band Edge @ f_s/R
Solution 1	110	0.14 (≤ 0.05)	27.3 (≥ 27)	92 (≥ 90)	0.42 (≤ 0.25)
Solution 1	160	0.072 (≤ 0.05)	26.3 (≥ 27)	91.7 (≥ 90)	0.42 (≤ 0.25)
Solution 2	110	0.048 (≤ 0.05)	91.5 (≥ 90)	91.5 (≥ 90)	0.2 (≤ 0.25)

Note to Table 2:

(1) Values in parentheses are required values for the parameter.

Conclusion

This application note presents design considerations for CIC compensation filters in a sample rate conversion system. Although CIC filters can implement decimation and interpolation efficiently in hardware for a wide range of rate change factors, a CIC filter response lacks a flat pass band response and good transition bandwidth. To alleviate these problems, a compensation FIR filter can be designed to provide frequency correction and spectrum shaping, even with an additional rate change of factor 2. This application note demonstrates how to use the MATLAB Signal Processing Toolbox to design a compensation filter using the frequency sampling method `fir2`. A sample script is given for reference. Detailed examples using WiMAX digital down conversion parameters are shown to illustrate how to choose parameters for the compensation filter and CIC and FIR filter cascade.

Document Revision History

Table 3 shows the revision history for this application note.

<i>Table 3. Document Revision History</i>		
Date and Document Version	Changes Made	Summary of Changes
April 2007 v1.0	Initial release.	—



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