INTRODUCTION

The Intel® Math course is a scaled-up adaptation of the Vermont Mathematics Initiative (VMI), a content-intensive professional development program developed by Dr. Kenneth Gross, Professor of Mathematics and Education at the University of Vermont. Intel Math provides eighty hours of professional development in mathematics for K-8 teachers (teacher participants), in the form of a course co-facilitated by a practicing mathematician and a mathematics educator (instructors). The course "is designed to close the gap between insufficient mathematics training of elementary school teachers and the demands of the contemporary mathematics classroom" (*Kenneth Gross*, on VMI[i]) and places emphasis on deepening the teacher participants' understanding of core K-8 mathematics concepts. It is grounded in a problem-solving approach to topics such as integer arithmetic, the decimal number system, place value, rational number arithmetic, rates, linear equations, and functions. About 90% of the course is focused on mathematics content knowledge and the remaining 10% on pedagogy.

BENEFITS FOR TEACHERS AND THEIR STUDENTS

> *"Deepening teachers' content knowledge is a priority for districts since research suggests that students are disadvantaged, and actually learn less, when their teachers do not understand the content (Goldhaber & Brewer, 2000; Monk, 1994). Progress in rectifying this situation stands to be a major contribution of Intel Mathematics."* (*WestEd evaluation report, 6/30/2009.*)[ii].

Evaluation results (*WestEd evaluation report*, 6/30/2009)[ii] indicate the Intel® Math course "provides teachers with opportunities to deepen their content understanding and to consider pedagogical issues related to their roles as mathematics teachers". In particular, teachers who completed the course have demonstrated growth in mathematics, in both their computational skills and their conceptual understanding. They have commented on varied ways of applying knowledge gained from the course to their classrooms, have indicated an increased focus on communication and reasoning, and reported multiple benefits to their students. Lasting benefits mentioned by teachers include "mathematics knowledge and strategies, access to resources, increased confidence in mathematics learning, changes in approach to teaching mathematics, an appreciation for the importance of making connections across mathematics topics, and understanding student thinking and learning styles".

COURSE CURRICULUM

The course is organized into eight units, each of which is comprised of 4 to 7 sessions. Intel Math teacher participants receive the two-volume *Teacher Manual*, in which each session focuses on mathematics content through a series of problems. Additionally, teacher participants receive an *Answer Manual*, which gives multiple solutions to each problem, and a *Reference Manual*, which expands on the philosophy and themes of Intel Math and offers supplemental readings.

Instructors receive all the materials that the teacher participants receive, and an additional *Instructor Manual*, which is essentially a session-by-session companion to the *Teacher Manual*. The *Instructor Manual* also includes a 30+ page course introduction that details aspects essential for a successful implementation of Intel Math, true to its philosophy and goals. A section on manipulatives, for example, discusses how the latter may be useful tools for representing and generating ideas, yet also highlights that manipulation of didactic physical objects may not, in and of itself, elicit or unpack conceptual thinking. Specific guidelines on how to select teachers to present their solution strategies and how to organize the order in which solutions are

shared are also included as part of the discussion on "Facilitating Presentation of Answers" (pg. 20). Additionally, the program allows for differentiation of the materials and homework assignments to meet teachers at various levels of math content knowledge. The complete two-volume *Teacher Manual* table of contents is reproduced below.

UNIT 1: ADDITION

- Different methods of solution
- Interconnectedness of arithmetic, algebra and geometry
- Meaning of "equals"
- Adjective-noun theme for addition
- Different ways of solving problems
- Solving simple equations
- Pedagogy: Student Understanding of Addition
- Supplemental Problems

UNIT 2: SUBTRACTION

- Properties of the number systems
- Meaning of subtraction
- Adjective-noun theme for subtraction
- Alternate algorithms for subtraction
- Processes and inverse processes
- Addition and subtraction of signed numbers
- Pedagogy: Student Understanding of Subtraction
- Supplemental Problems

UNIT 3: MULTIPLICATION

- Meaning of multiplication
- Adjective-noun theme from multiplication
- Distributive property
- Area model for multiplication
- Multiplication of signed numbers
- Primes and Composites
- Least Common Multiple and Greatest Common Factor
- Problem-solving with LCM and GCF
- Pedagogy: Student Understanding of Multiplication
- Supplemental Problems

UNIT 4: DIVISION

- Meaning of division
- Models for division
- Adjective-noun theme for division
- Types of division - partitive and quotative
- Introduction to rates
- Pedagogy: Student Understanding of Division
- Supplemental Problems

UNIT 5: OPERATIONS WITH FRACTIONS

- Meaning of Fractions
- Add and Subtract
- Multiplication
- Fractions in Context
- Pedagogy: Student Understanding of Fractions
- Making sense of Fractions
- Supplemental Problems

UNIT 6: RATIONAL NUMBERS

- Exponents
- Decimals
- Algebraic Fractions
- Rates Revisited
- Rates in Context
- Pedagogy: Student Understanding of Place Values
- Supplemental Problems

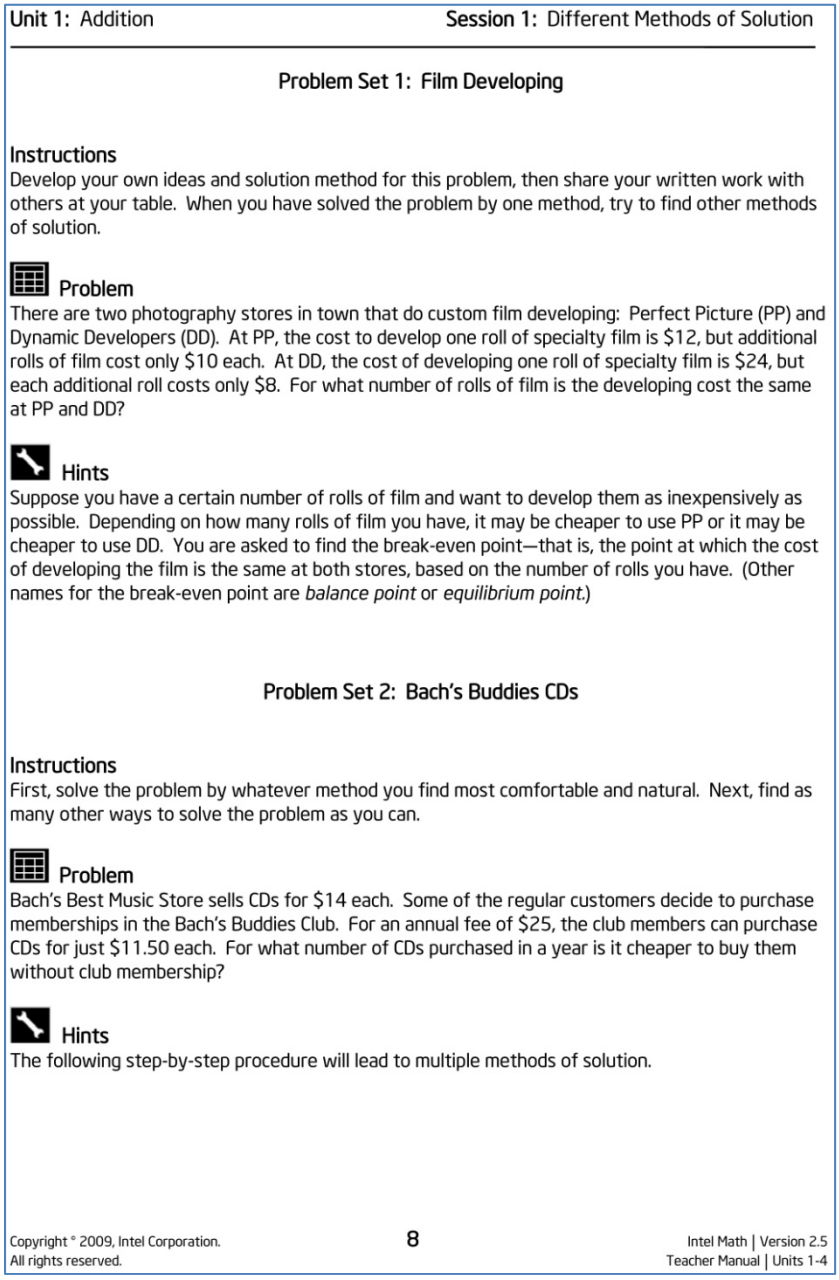
UNIT 7: LINEAR RELATIONS

- Feet and Inches
- Everyday Examples
- Slope of a Line
- Pedagogy: Student Understanding of Linear Equations
- General Linear Problems

UNIT 8: FUNCTIONS

- From Processes to Functions
- Function Features
- Linear Functions
- Functions in Context

As an example, below is a copy of the first page of the first Problem Set in Unit 1, which is focused on integer addition. Within the units are several sessions devoted to analyzing actual students' solutions. Most units close with a session containing supplemental problems.



Unit 1: Addition **Session 1:** Different Methods of Solution

Problem Set 1: Film Developing

Instructions
Develop your own ideas and solution method for this problem, then share your written work with others at your table. When you have solved the problem by one method, try to find other methods of solution.

Problem
There are two photography stores in town that do custom film developing: Perfect Picture (PP) and Dynamic Developers (DD). At PP, the cost to develop one roll of specialty film is $12, but additional rolls of film cost only $10 each. At DD, the cost of developing one roll of specialty film is $24, but each additional roll costs only $8. For what number of rolls of film is the developing cost the same at PP and DD?

Hints
Suppose you have a certain number of rolls of film and want to develop them as inexpensively as possible. Depending on how many rolls of film you have, it may be cheaper to use PP or it may be cheaper to use DD. You are asked to find the break-even point—that is, the point at which the cost of developing the film is the same at both stores, based on the number of rolls you have. (Other names for the break-even point are *balance point* or *equilibrium point*.)

Problem Set 2: Bach's Buddies CDs

Instructions
First, solve the problem by whatever method you find most comfortable and natural. Next, find as many other ways to solve the problem as you can.

Problem
Bach's Best Music Store sells CDs for $14 each. Some of the regular customers decide to purchase memberships in the Bach's Buddies Club. For an annual fee of $25, the club members can purchase CDs for just $11.50 each. For what number of CDs purchased in a year is it cheaper to buy them without club membership?

Hints
The following step-by-step procedure will lead to multiple methods of solution.

IMPLEMENTATION MODEL

The recommended implementation model consists of five to eight consecutive days during the summer and five to eight days dispersed throughout the fall semester. Each day is seven hours long, consisting of six hours of instruction, and one hour of break time. However, participants in previous offerings of the training program suggested limiting the amount of course time during the school year, so as to avoid taking the teachers away from their classes. Each site should therefore consider an implementation model that suits its particular cohort of teachers. Arrangements such as a full summer course, a summer course with minimal school year follow up, weekends, or other options may be considered.

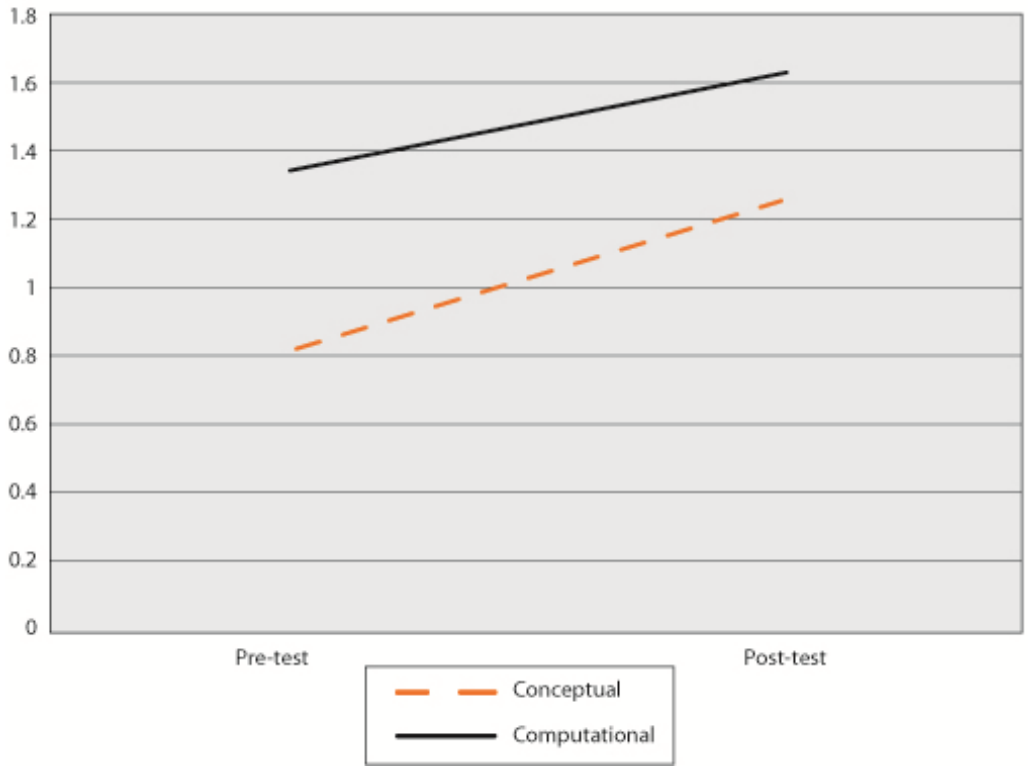
It is our recommendation that to maximize the effectiveness of the Intel Math program, a Mathematics Learning Community (MLC) component be added to helps teacher participants revisit and better implement what they've learned in their classroom. UMASS Medical School has developed materials for the MLC, to foster a community among teacher participants, for teachers to support one another as them implement what they've learned in the Intel Math program. For more information, visit www.umassmed.edu/Math_Learning_Community/index.aspx

EVALUATION OF PROGRAM

The following graphs and quotes are taken from the WestEd evaluation report (6/30/2009)[iii].

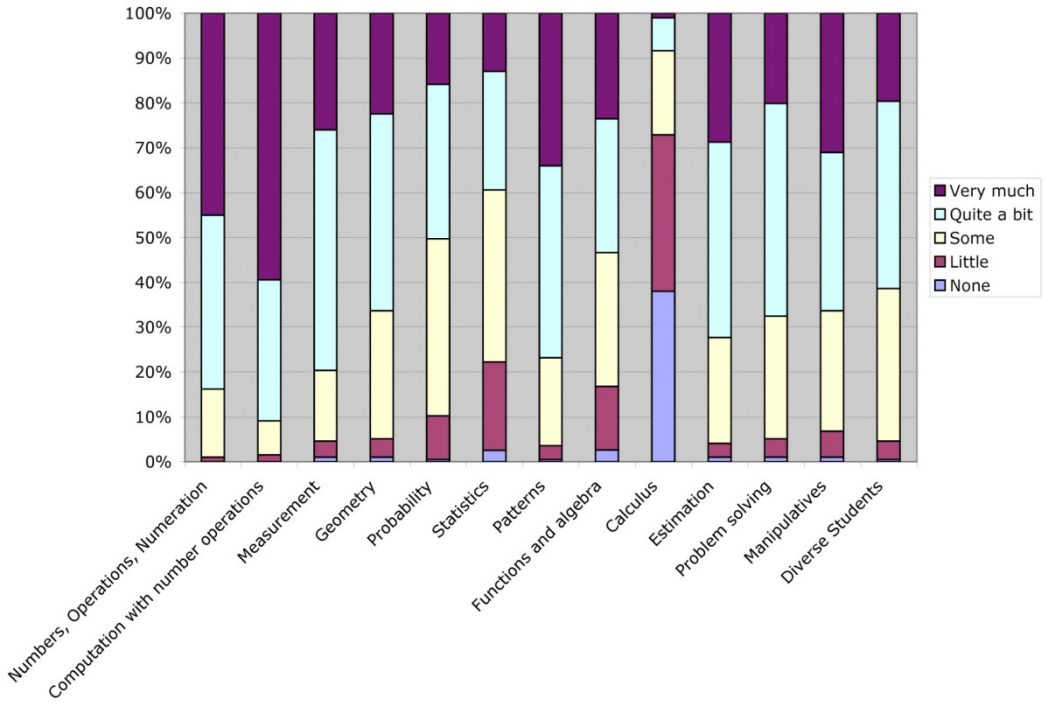
> *Graph 4 shows that teachers' scores increased both conceptually and computationally from pre- to post-test. The solid green line shows the difference between pre- and post-test on computational items. The dotted orange line shows the difference pre/post on conceptual items. While teachers' scored lower conceptually both pre- and post-course, the gap between the mean scores on conceptual and computational items decreased*[iv].

Graph 4: Growth in Conceptual and Computational Items on IM Course Survey

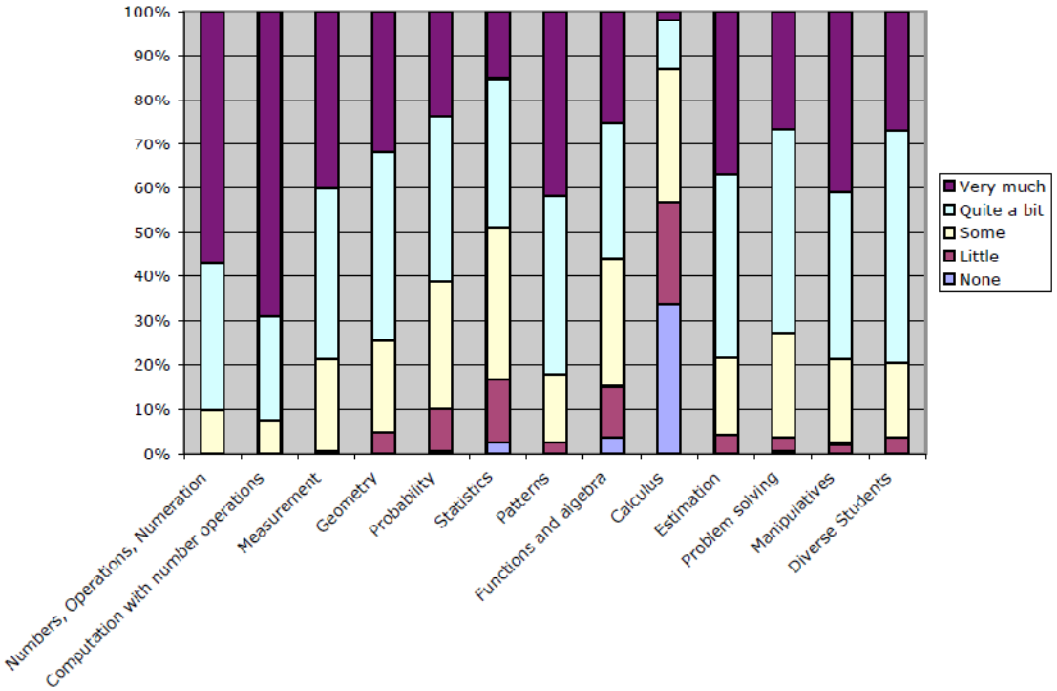


Teachers also rated how well prepared they felt in mathematics topics prior to and after completing the course. Graphs 7.1 and 7.2 report the teachers' responses on the pre-survey and post-survey respectively[v]. Note the gains made with regard to those topics included in the course, namely, "Numbers, Operations and Numerations," "Computations with number operations," "Geometry," "Patterns," "Problem Solving," and "Diverse Students."[vi]

Graph 7.1: Pre-Survey Teachers' Reported Level of Preparation (N = 202)



Graph 7.2: Post-Survey Teachers' Reported Level of Preparation (N = 202)



SELECTED QUOTES FROM K-8 TEACHER PARTICIPANTS

The following quotes were collected by WestEd as part of their 2008-2009 evaluation report[vii].

IMPROVED CONTENT KNOWLEDGE

- "I feel more comfortable with my understanding of math and also in how I teach it to kids."
- "It helped me renew my knowledge of mathematics. While I have learned all this at one point, one tends to forget but it is important to keep it all fresh for a better and deeper knowledge."
- "I have always loved doing math, but have not had the opportunity to take a course for a couple years now. The course brought to light a lot of different ways to solve problems. I really enjoyed the course and learned a lot of new techniques."

IMPROVED PEDAGOGY

- "I am far and away a better math teacher than I could possibly have been otherwise."
- "I can teach students on their level instead of teaching to the middle of the class."
- "My students have been able to learn mathematics in new ways and they are actually more excited about math."
- "This has helped my students because I now look at errors very differently. I look for what they did right first and plan from there."
- "I am always asking questions to see how the students are thinking."
- "I am able to ask better questions so that they can use prior knowledge to get to the next step without me telling them."

PERSPECTIVES ON THE PROGRAM

- "[Intel Math] helped me to present the math curriculum content with a deeper understanding of the math learning experience/struggle that my students go through."
- "It helps me see the big picture, as to what do my students need in the future, what do they need to know to succeed … It also helps me see the bigger math picture. Math skills often are so fragmented on each grade level that it is often difficult to see what each one of them is leading up to and in what way it is a building block for future mathematics. This course helped clear this up."

ROLES

TEACHER PARTICIPANTS

Teacher participants commit to complete all 80 hours of the course, and to complete regular homework assignments. In past implementations, sites have chosen to reward teacher participants for completion of the Intel Math program in a variety of ways. Ranging in value from $1,000 to $2,000, incentives have been in the form of cash stipends, college credit, document cameras, or a combination thereof.

INSTRUCTORS

Approved instructors commit to a four-day certification workshop, and to co-teach two 80-hour courses within two years (i.e. one course a year for two years). Instructor compensation is typically equivalent to

approximately seven weeks of pay, but is localized as instructors are contracted through the local site implementing Intel Math. Sites can subdivide instructor compensation into one week for training, two weeks for each course, and one week for planning.

Individuals interested in becoming instructors may apply through the Institute for Mathematics and Education website at ime.math.arizona.edu/intelmath/.

SENIOR TRAINERS

Senior trainers are exemplary instructors who have experience teaching Intel Math and have been selected by the Institute for Mathematics and Education to train other instructors. These senior trainers also form pair-teams consisting of one mathematician and one math educator. Senior trainers facilitate the four day intensive training for instructors.

INSTITUTE FOR MATHEMATICS AND EDUCATION

The Institute for Mathematics and Education (IM&E) at the University of Arizona manages Intel Math at the national level. IM&E handles the oversight of the recruitment, training, dissemination, implementation, and evaluation of the program. For a complete list of services and support the Institute offers, both for Intel Math and for the mathematics education community at large, please visit ime.math.arizona.edu.

PROGRAM COST

The cost of the Intel Math Program varies, depending on the number of teachers trained at each site, the localized precedents for incentives and stipends, and the scale to which the program is implemented. The following is a guideline for the breakdown of the cost of various elements of the Intel Math program:

INSTRUCTOR TRAINING

• Hosting Instructor Training (includes planning of event, instructor selection, Sr. Trainer stipends, materials)	35,000
• Instructors' stipends for training (includes cost of travel, accommodations)	40,000
Total	$75,000

Please note this guideline is based on hosting your own training for 20 instructors. It does not include the facility costs, if applicable. Most sites will not host their own instructor training. This breakdown is for states or sites who would like to implement Intel Math on a larger scale.

COURSE DELIVERY

• Instructor Training (includes cost of training, travel, and stipend for two instructors at training)	7,500
• Instructor Costs (includes stipends for two instructors, supplies for course)	21, 000
• Intel Math Course (includes printed materials, resources, assessments, evaluation, administrative and grant support from IM&E)	7,000
• Teacher Participant Stipends ($1,000/teacher)	30,000
Total	$65,500

This guideline is based on one course of 30 teacher participants, and includes the start-up cost of training new instructors for the site. Teacher and instructor stipends will vary by site and local precedents.

NEXT STEPS

If you are interested in implementing the Intel Math Program in your state or school district, we encourage you to:

- **Consider your team**. A successfully run program requires teams of two instructors (one mathematician, one math educator) for each class. Do you know mathematicians and/or math educators who you would like to recommend apply to be trained as Intel Math instructors?

- **Consider the scale**. How many participant teachers would you like to see participate in the program? How many instructors would you need? How many classes or locations?

- **Consider funding options**. How can you fund this professional development program? What grants are available through your state or federal government?

- **Contact us**. Email Aubrey Neihaus, Project Manager for Intel Math, at aneihaus@math.arizona.edu. We are eager to offer our help and guidance as you consider your site's implementation and strategy.

COLLABORATORS

- **Achieve –** supports interactions with States, especially with current Achieve work <www.achieve.org>
- **American Mathematical Society (AMS) –** supports instructor recruitment and program marketing <www.ams.org>
- **Arizona Department of Education -** implements the program within Arizona <www.ade.state.az.us/>
- **Intel Corporation and Foundation –** program founder and supporter <www.intel.com/education/>
- **Massachusetts Department of Elementary and Secondary Education –** implements program across MA www.doe.mass.edu/
- **Mathematical Association of America (MAA) –** manages the June 2010 instructor training and supports instructor recruitment <www.maa.org>
- **New Jersey Department of Education –** implemented the program at Montclair State College <www.state.nj.us/education/>
- **UMASS Medical School –** developer of Mathematics Learning Community component <www.umassmed.edu>
- **Silicon Valley Leadership Group –** business organization funded implementation in Bay Area, CA <svlg.org>

[i] http://www.emba.uvm.edu/~gross/index_files/Page399.html

[ii] The WestEd report assessed the following evaluation question: "To what extent do participating teachers increase their mathematics content knowledge?". It also considered "teachers' pre and post course attitudes and beliefs about mathematics teaching and learning". WestEd is a national nonprofit research and service agency.

[iii] The instrument used in Graphs 4, Graphs 7.1 and 7.2 of the WestEd evaluation is the Teacher Survey, adapted from the Vermont Mathematics Initiative teacher survey. © 1999-2007 Vermont Mathematics Initiative, Kiran S. MacCormick.

[iv] Page 13, *WestEd evaluation report (6/30/2009)*

[v] Page 17, *WestEd evaluation report (6/30/2009)*

[vi] Note that the following topics were not covered in the Intel Math course: Statistics, Functions and Algebra, and Claculus.

[vii] Participant Teacher quotes are taken from an online survey conducted by WestEd as part of the evaluation report (6/30/2009)